

Unit QUAN – Session 6

Introduction to Acceptance Sampling

MSc Strategic Quality Management Quantitative methods - Unit QUAN



INTRODUCTION TO ACCEPTANCE SAMPLING



Aims of Session

To introduce the basic statistical concepts of acceptance sampling, operating characteristic curves and their interpretation and use. (This session does *not* aim to provide a review of standard plans such as those in BS6001.)

Learning Approach

Study Notes, Required and Recommended Reading, Questions to Stimulate Your Thinking, Self-Appraisal Exercise,

There is also a case study (on an examination Board) which relates the content of the unit to a practical example. You should work through the exercises in this and compare your answers to those suggested.

Content

- ! Acceptance sampling.
- ! Sample selection.

- ! Operating characteristic curves and their interpretation.
- ! Producer's and consumer's risk.
- ! Design of suitable single sampling schemes for attribute data.

The session also draws on Tables 1 and 2 in the Tables at the end of this unit.

Software

Many spreadsheets (e.g. Microsoft Excel) and other packages have built in functions for the binomial and Poisson distributions - check the Help button.

There is some software available from <http://woodm.myweb.port.ac.uk/programs.htm> which you may find helpful.

Reading

You will find further reading on this topic in Besterfield (2004), Chapters 9 and 10 (please see the book list in the Preamble for full details of this book).

April 2010.

INTRODUCTION TO ACCEPTANCE SAMPLING

Acceptance Sampling

Acceptance sampling refers to the process of taking a sample from a batch of items in order to come to a decision about whether or not to accept the batch. Acceptance sampling is, in a sense, a method of last resort, since it is far better to control the process producing the items rather than rejecting a batch which has already been produced. In addition, sampling the end product is generally unable to guarantee the very high levels of quality which are now required. For these reasons, it is an unfashionable topic which is not mentioned in many modern books on quality. However, there are some situations in which acceptance sampling is useful.

I will start with a summary, and then the questions below are designed to take you through some examples of how it works in practice—it is important that you work through the questions. We will build on the Binomial and Poisson distributions which were covered in an

earlier session. You may find it helpful to use a computer for the calculations (see the note on software above); alternatively there are some Poisson tables at the end of the unit.

A **sampling scheme** (or plan) is simply a statement about how and when samples should be taken, and what action should be taken on the possible outcomes. The simplest type of plan - a single sampling plan - specifies the results which lead to accepting the batch and the results which lead to rejecting the batch. For example, a scheme might specify taking a **random** sample of 50 items and accepting the batch if 0 or 1 are defective, but rejecting it for 2 or more defective items.

Sometimes it is convenient to have a short way of describing such a scheme—we will use the notation 50, 1 or 50, 1, 2 to describe this scheme. The first number refers to the size of the sample; the second number is the acceptance number—the greatest number of defects in the sample which leads to acceptance of the batch, and the third number is the rejection number—the least number of defects which leads to rejection. With single sampling plans we don't really need the third number because if the batch is not accepted it must be rejected, but this is not true of more complex schemes involving *double*, *multiple* or *sequential* samples since in these there may be a range of results which will lead to a further sample being taken. In general the more complex schemes are more efficient in the sense that the same results can be achieved by inspecting fewer items. On the other hand, they are more complex to administer. Here we will just consider single schemes.

Once we have decided to set up an acceptance sampling system, the problem, of course, is to decide what is the best sampling scheme to use. The way we go about this is very simple in principle, although the details can get complicated. The idea is to consider *each possible sampling scheme* and work out what the effect of the sampling scheme will be for *all possible quality levels in the batch*.

The results are conveniently shown on an **operating characteristic (OC) curve** for each sampling scheme. This is a graph showing the probability of the sampling scheme in question accepting a batch, against the actual proportion defective in the batch (or another measure of the quality of the batch). Figure 1 shows an example. As we would expect, if the proportion defective is 0%, the probability of accepting the batch is 100%. This probability gradually declines as the proportion defective in the batch increases. In rough terms this should be what we expect.

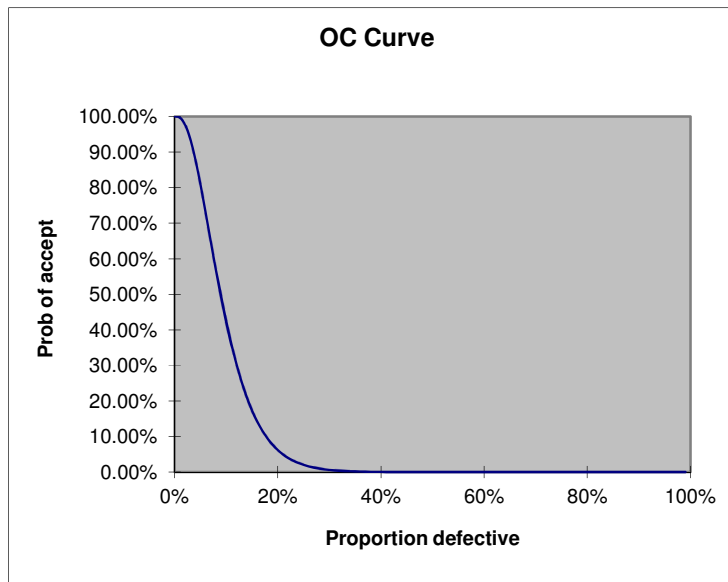


Figure 1: OC curve for the sampling scheme 30, 2.

These curves can be calculated using the binomial or Poisson probability distributions. The binomial¹ distribution is appropriate when the quality level is measured in terms of the proportion of defective items, and the Poisson distribution is appropriate when the measure is defects per unit (item) or 100 units. (There may be more than one defect in a unit.) However, the Poisson distribution is easier to use and tabulate and can be used as a reasonable approximation to the binomial distribution if the proportion defective is less than 10% - which, we hope, it usually is. For this reason we will use the Poisson distribution here.

You will find tables of Poisson probabilities with the tables at the back of this unit. To use them you need to work out the mean (average) number of defects (or defectives) per sample. For example, if the sample size is 30 and you are interested in the situation when 2% of the batch is defective (i.e. $p = 0.02$) then the mean number of defectives per sample is $0.02 \times 30 = 0.6$. Some samples will have no defectives, some will have one, some will have two, and so on, but the overall average will be 0.6

¹ Strictly, the binomial distribution can only be used when batch size is infinite; otherwise the hypergeometric distribution should be used - see Mitra (1993). However, in practice, provided that the sample size is much less than the batch size - say less than 25% - the binomial distribution provides a reasonable approximation. The spreadsheet at <http://woodm.myweb.port.ac.uk/occurve.xls> allows you to compare the three distributions.

Next, to take the sampling scheme 30,2 as an example (Figure 1), we need the probability of the sample containing either 0 or 1 or 2 defectives—these are the results that will lead to acceptance of the batch. Table 2 gives this probability as 0.9769 or 97.69%. (Look along the top for mean = 0.6. Then you will find the probability under the heading “r OR LESS” along the r = 2 row. Alternatively you can use the top column of figures to work out the probability that r is exactly 0 or 1 or 2 and then add them up.)

To produce Figure 1, all you need to do is to work out the equivalent probability for 0%, 1%, 2% (defective in the batch) and so on, and join the dots up. This is how Figure 1 was drawn.

Having drawn an OC curve for a particular sampling scheme, you can then examine it to see whether the scheme does what you want it to. Does it reject batches that you would want rejected and accept those you would want accepted? Would another sampling scheme be more suitable?

Each sampling scheme is represented by one line or OC curve, so if we want to decide on the best sampling scheme, we need to draw one for all the possibilities. For example, if you have a look at the suggested answer to Questions 4 and 5 below, each line on this graph represents a different sampling scheme. And there are obviously many more possibilities! This is a complicated choice, so we need a way of formalising it.

The choice of a suitable sampling scheme can be formalised by using the concepts of “AQL” and the consumer's risk, and “LQL” and the producer's risk.

The **acceptable quality level (AQL)** is a "good" quality level which is acceptable to the consumer - typically stated in a contract or purchase order. The probability that the sampling plan will *reject* a batch with this acceptable quality level (AQL) is called the **producer's risk** (α) because it corresponds to the risk that a producer producing output at the acceptable quality level will, nevertheless, have the output rejected. The producer would, of course, like this level to be as small as possible. The conventional maximum level of the producer's risk is 5%.

The **limiting quality level (LQL) or lot tolerance proportion defective (LTPD)** is an unacceptable level of quality for the consumer. The probability of *accepting* a batch with this level of quality is called the **consumer's risk** (β), because it corresponds to the risk that the consumer will end up receiving output at this unacceptable quality level. The consumer

would, of course, like this risk to be as low as possible. The conventional maximum level of the consumer's risk is 10%. (This convention seems rather dated now, as it implies the consumer should be subjected to a higher risk than the producer!)

In practice there is inevitably some arbitrariness about the AQL and the LQL. If, say, the AQL is 2% and the LQL is 8%, then obviously 1% would be entirely acceptable (as it's better than the AQL), and 10% would be even more unacceptable than the LQL. The AQL and the LQL are simply two quality levels chosen for the purpose of assessing how a sampling scheme will work.

Let's see how this works if the AQL for a particular batch is 2% and the LQL is 8%. We'll now see how the sampling scheme in Figure 1 fares on these criteria. The producer's risk is the probability of a batch of quality level 2% being rejected. We worked out above that the probability of such a batch being accepted is 97.69%, so the probability of it being rejected is 2.31%. This producer's risk is less than 5% so it is acceptable. The sampling scheme is OK from this point of view.

The consumer's risk is simply the probability of a batch of quality level 8% being accepted—using exactly the same method as above, the mean number of defectives in a sample is 2.4, and the Poisson distribution then tells us that the probability of such a batch leading a sample with 2 or less defectives is 56.97%. These results are shown in Figure 2. In this figure the length of the dotted vertical line represents the consumer's risk as a probability measured on the vertical scale, and the shorter line on the top right represents the (smaller) producer's risk.

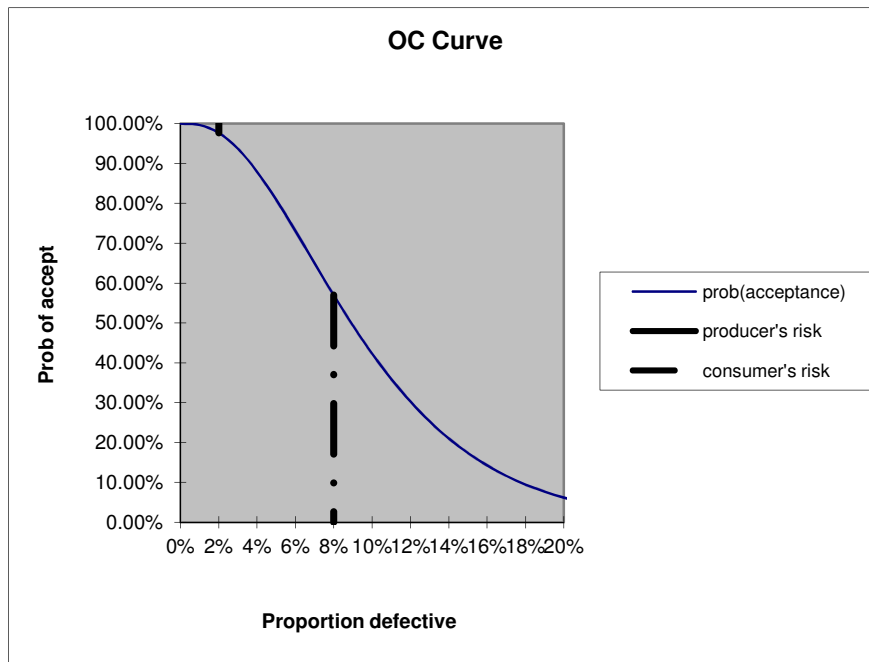


Figure 2: OC curve showing producer's risk and consumer's risk

This consumer's risk is too high—far higher than the 10% conventional requirement, so the sampling is not satisfactory. We must try to find another, more suitable, one.

In principle, the way to this is very simple: make a list of all possible schemes, starting from the simplest (smallest sample size), and then work through the list until a suitable scheme is found. In practice we can either do this with a computer program (there is such a program at <http://woodm.myweb.port.ac.uk/accsamp.exe>), or use tables. Table 1, at the end of the unit, can be used for this purpose. Remembering that the AQL is 0.02, and the LQL is 0.08, this table indicates that the most suitable sampling scheme is 98,4—i.e. take a random sample of 98 and accept the batch if there are 4 or less defectives in the sample. Any smaller sample will fail to achieve a satisfactory producer's or consumer's risk.

This is just an overview. It is important now to work through the questions below to see how it works in more detail.

QUESTIONS TO STIMULATE YOUR THINKING



Having read the study notes on "**Introduction to Acceptance Sampling**" now consider and answer the following questions, the purpose of which is to show you how to apply the information given in the course notes for this session.



Questions

- 1 Why is it important that samples should be random?
- 2 What is the probability of the sampling scheme 40,1 (i.e. sample 40 and accept if the number of defects found is 0 or 1) leading to the acceptance of a batch with 2% defectives?

To do this you need to:
 - (i) work out the mean number of defectives in a sample of 40 from such a batch, and then
 - (ii) use the Poisson distribution to calculate the probability of the batch being accepted. (You may use Table 2 attached for the Poisson distribution if you need to.)
- 3 Work out the probability of the scheme 40,1 accepting batches with defect rates:
 - (i) 0%
 - (ii) 1%
 - (iii) 10%
- 4 Now use the results from questions 2 and 3 to sketch the OC curve for the sampling scheme 40,1.

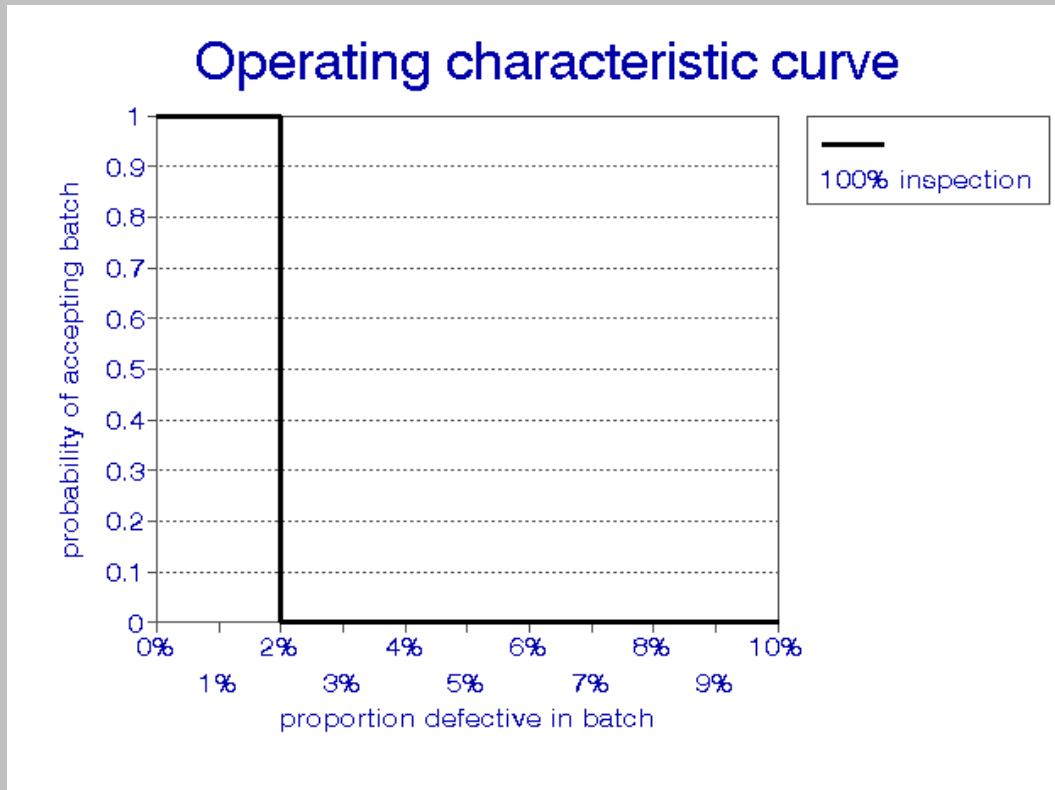
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- 5 Sketch the OC curve for the following sampling plans:
- (i) 40,2 (i.e. sample of 40, accept if up to 2 defects)
 - (ii) 10,0
 - (iii) 10,1
 - (iv) 10,2
- 6 Sketch the OC curve for a scheme which says "check all items and reject if 2% or more are defective".
- 7 A sampling plan calls for a sample size of 50 and an acceptance level of 1. The contract calls for an acceptable quality level of one defective item 100 and the lot tolerance proportion defective (limiting quality level) is six defective items 100.
- What are the producer's and consumer's risk for this plan?
- 8 What criteria would you suggest for setting the AQL, LQL, producer's risk and consumer's risk? Give examples.
- 9 Use Table 1 to find a sampling scheme for an AQL of 1.4%, LQ of 6%, producer's risk 5%, consumer's risk 10%. Use the Poisson distribution (or a computer or calculator) to check this scheme. (You will need a value which is not in the tables: the Poisson distribution with a mean of 7.9 gives the probability of 4 or less events as 10.5%.)
- 10 The sampling scheme 66,2 has the same ratio of acceptance number to sample size as your answer to Q9. It is also a smaller sample and more convenient from this point of view. Does this scheme meet the criteria in Q9?
- 11 Use Table 1 to find a sampling scheme for an AQL of 1.5%, LQ of 6%, producer's risk 5%, consumer's risk 10%. (1.5% is not one of the AQLs tabulated in Table 2 - so you need to decide whether to take a slightly higher or lower AQL in the table.)

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- 6 Obviously, if the level of defectives in the batch is less than 2% the batch will be accepted, if it is more the batch will be rejected. This means the OC curve will have a value of 1 or 100% (on the y / probability axis) from 0% to 2% on the x axis, and 0 above 2%. It is a "step function" and not an elegant curve.



This sampling plan discriminates perfectly between quality levels better than 2% and those which are worse than 2%. The cost, however, is 100% inspection.

- 7 For a batch with the AQL level of quality, the mean number of defectives per sample of 50 is 1% of 50 or 0.5, and the probability of such a batch being accepted by the sampling scheme is 91%. This means that the probability of its being rejected is 100%-91% or 9% - which is the producer's risk. Similarly an LQL batch has a mean of 6% of 50 or 3 defectives per sample, which means the probability of the batch being accepted (the consumer's risk) is 20%.

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8 The AQL should be a level of quality which is acceptable to the consumer, and the LQL is a level which is not acceptable. In practice the choice of these two levels is to some extent arbitrary: the important thing is that the producer's risk is defined in terms of the AQL, and the consumer's risk is defined in terms of the LQL. These two risks should reflect the seriousness of the two kinds of error. If, for example, acceptance of an LQL batch would be a very serious matter for the consumer (perhaps the items in the batch are critical components of some medical apparatus) then the consumer's risk should be set at a very low value.

9 The scheme given by Table 1 is 132,4. For this scheme:

Producer's risk = 3.6% Consumer's risk = 10.5%

(Using Table 2 and rounding means to the nearest number given.)

At first sight this is not a suitable scheme because the consumer's risk is slightly too high. However, the table uses the binomial distribution which gives a consumer's risk of 9.7%.

10 The producer's risk is 8% and the consumer's risk is 24% (rounding the means to the nearest value given in Table 2). This means it meets neither of the risk targets (5% and 10%): the sample is too small to discriminate between batches with sufficient accuracy.

11 If we take an AQL slightly higher than the value given in the table, the probability of the batch being rejected - the producer's risk - will obviously be greater (because there are more defectives). This means it may fail to meet the 5% criterion. If, on the other hand we take an AQL slightly lower, the probability of the batch being rejected will be less. This means that the scheme for an AQL of 1.4% should meet the criteria: 132,4 as in Q9. For this scheme:

Producer's risk = 4.4% Consumer's risk = 10% (as before)

(Using Table 2 and rounding means to the nearest number given.)

(In fact a computer search reveals that 132,4 is the most efficient sampling scheme for this situation too.)

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SELF-APPRAISAL EXERCISE

Exercise



You will have done the questions for **session 1** of this unit, and absorbed the messages which the session's material contains. Here follows an exercise to give you further information and food for thought.



The assigned task is:

- 1 Acceptance sampling schemes are now unfashionable. What alternative strategies are available to guarantee quality? In what circumstances do you think acceptance sampling schemes still have a role to play?
- 2 Ideally sampling schemes should sort good batches from bad samples without making any errors. Suppose that a 5% level of defectives is acceptable, but that anything over that is not? Is it possible for a sampling scheme to discriminate with complete accuracy between good batches and bad ones? How would you go about designing a suitable sampling scheme in practice?



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TUTORIAL COMMENTARY



- 1 The problems with acceptance sampling schemes are that they are geared towards accepting quality levels which are less than perfect, and they are a reactive "after the event" approach rather than a proactive way of ensuring that the quality is acceptable.

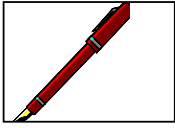
The main alternative strategy is to use SPC, or something similar, to ensure that the process is capable so that it is not necessary to check each batch.

Acceptance sampling schemes may still have a role to play where it is not possible to be sure of the capability of the process—for example in the case of processes where there are unavoidable uncertainties which make the quality of the output unpredictable, or in the case of a new supplier whose quality procedures are unknown.

- 2 No sampling scheme can discriminate with perfect accuracy between good batches and bad. Only 100% inspection can do this. In general, the larger the sampling scheme the better the discrimination.

One approach is to take an AQL of, say, 4%, and an LQL of, say, 6%, and then find a sampling scheme which gives satisfactory values of the producer's and consumer's risks.

Alternatively you could plot the OC curves of possible sampling schemes and then examine them to see if they treat batches of different quality levels in a satisfactory manner.



Examination Board Case Study

This case study concerns a newly formed Examination Board created by the merger of several smaller boards that had been in existence for a long time. The introduction of a new examination in 2000 stretched the organisation to breaking point and resulted in a lot of adverse comment in the press and some extreme political posturing when the first results were published in 2002.

The Qualifications and Curriculum Authority (QCA) had conducted an audit during 2001 which focused on 3 main areas;

- The management structure
- Information and communication systems
- Quality assurance and quality control

As a result of the audit a series of recommendations were made. Political pressure resulted in the audit report being published earlier than planned and the QCA assigned its own director of quality audit to oversee the improvements required.

Recently the CEO of the Examination Board published an article explaining the process for ensuring the accuracy and quality of the final marks. The key points from the article are;

This year 85 million questions on 5 million papers will be marked. A multi-layer quality control system was used in the past;

1. All examiners are trained in assessing standards and in marking papers. They must double check each paper that they mark.
2. A 100% clerical check is performed to ensure that all questions have been marked and all marks added up correctly.
3. A further check is conducted, but only on request from the candidate or the school.

Two years ago, as an improvement to the quality control process a sampling method was introduced as a better approach than the 100% clerical check that had been conducted. An analysis of the results of the 100% clerical check had revealed that it was only detecting 8 out of 10 marker errors. The explanation was that 100% inspection regimes can be poor error detectors because of limitations in human observation. This reflected the fact that people get tired and distracted, especially when you are using 700 temporary clerical staff to recheck 5m scripts over a 4 week period (which roughly equates to 1 minute per script!).

A new process has now been implemented;

- a. Clerical assistants check a random sample of 10 scripts from each batch for errors. (Scripts are in batches of 500 marked by the same examiner.)
- b. If *any* errors are found, all scripts from that examiner are checked.

This sampling approach is claimed to lead to accuracy levels of more than 99% - i.e. it will lead to less than 1% of scripts having clerical errors.

(This process only deals with clerical errors. Other marking errors – e.g. examiners applying the assessment criteria incorrectly – are not detected by this checking process.)

Exercise A

What is the probability of this sampling approach leading to the acceptance of a batch of scripts 1% of which have clerical errors?

What is the probability of this sampling approach leading to the acceptance of a batch of scripts 5% of which have clerical errors?

What is the probability of this sampling approach leading to the acceptance of a batch of scripts 20% of which have clerical errors?

Are these conclusions satisfactory?

Do you think it is sensible to claim that a sampling scheme like this will lead to a specified level of accuracy?

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A better sampling scheme would entail a larger sample size. We can use the theory in the notes to devise a suitable scheme.

Acceptance sampling is the process of evaluating a portion of the product in a batch for the purpose of accepting or rejecting the entire batch. In our case the product is a marked examination script and a batch is a batch of scripts marked by a single examiner.

Reasons for using acceptance sampling are

1. the cost of inspection is high in relation to the damage cost resulting from passing a defective product.
2. 100% inspection is monotonous and causes inspection errors.
3. The inspection is destructive.

In this case, reason 2 certainly applies; 100% inspection is only detecting 80% of marker errors. This is in part due reason 1 - the checkers only have an average of 1 minute to check each script because the cost of having more checkers (there are already 700) is too high.

Our sampling plan should be designed to minimise the risk that bad batches of scripts can be accepted. The effect on the consumer (the A level student) of a bad lot being accepted is that an inaccurate mark may cause an erroneous grade to be awarded, which in turn could result in the student being denied a place at university.

The *acceptable quality level* (AQL) is the level acceptable to the consumer. For this we can take the CEO's quoted figure of 99% defect free or 1% defective. The probability that the sampling plan will reject a batch with this AQL is the *producer's risk*, which is conventionally 5%. The *limiting quality level* (LQL) is the unacceptable level of quality for

the consumer. For this we will assume 5% defective as a starting point. The probability of accepting a batch with this level is the *consumer's risk*, which is conventionally 10%.

Exercise B

1. Given the information provided above, devise a **single sampling** plan that could achieve these criteria.
2. Create the relevant Operating Characteristic Curve. How do you interpret the level of risk of accepting batches containing wrongly marked examination scripts? Do you consider the level of risk to be acceptable?
3. Have you any general comments on the use of acceptance sampling here?

Examination Board Case Study – Answers

Exercise A

If 1% have clerical errors, the mean number of errors in a sample of 10 is 0.1. The Poisson tables then indicate that there is 90.48% probability that the sample will have no scripts with errors – so this is the probability that the batch will be accepted without further checking.

Using a similar method, the probability of accepting (without further checking) the batch with a 5% error rate is 60.65%, and the probability of accepting the batch with a 20% error rate is 13.53%.

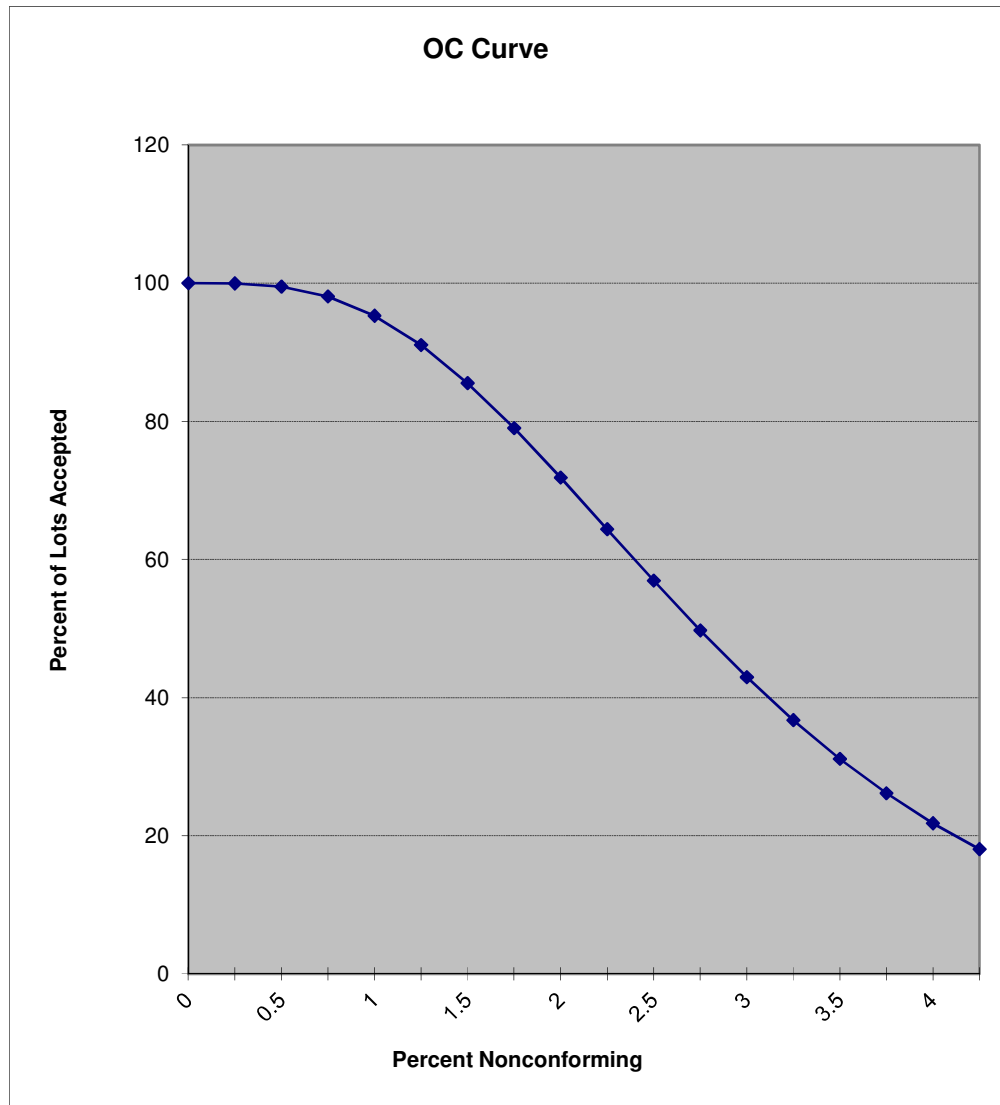
These results are obviously not at all satisfactory. A 13.5% probability of accepting a batch of 500 scripts 100 of which contain clerical errors is obviously unacceptable!

The main factor leading to clerical accuracy is the care exercised by the examiners. The sampling scheme can help by ensuring that batches with high levels of errors are detected and checked thoroughly. However, as the results above show, a sample of 10 is too low to achieve this consistently. Whether the overall error rate is less than 1% depends on the number of examiners with high error rates – which is not influenced by the sampling scheme.

Exercise B

- 1 Using Table 1 at the end of the unit, an appropriate *single sample scheme* would be ***a sample size of 132, and accept with 3 or fewer defective.***

2. Create the relevant Operating Characteristic Curve



The purpose of an Operating Characteristic Curve is to help with the judgement about whether a particular sampling plan will be suitable – it evaluates the effectiveness of a particular sampling plan. It tells us the probability that a batch submitted with a certain percentage nonconforming (defective) will be accepted. If the sampling plan is not

satisfactory, as shown by the OC curve, another one should be selected and its OC curve constructed.

This chart tells us, for example, that

- a) There is a 100% probability of a lot being accepted that has 0 % defective (obviously!).
- b) There is an 18% probability of a lot being accepted that has 4.5% defective.
- c) There is a 95% probability of a lot being accepted that has 1% defective.
- d) There is an 80% probability of a lot being accepted that has 2% defective.

And so on.

The question now is - given the sensitivity of allowing incorrectly marked examination scripts to affect candidate's grades, is this level of risk acceptable?

2% defective on a lot size of 500 would result in 10 incorrectly marked scripts being accepted. An 80% probability of this occurring would seem to be unacceptably high and another sampling plan should be devised.

The difficulty here, of course, is that the sample is already 132 of the batch of 500 scripts. A better scheme is likely to approach 100% inspection. The most effective way forward is likely to be to look at the causes of the errors, and at ways of eliminating them. As this analysis shows, acceptance sampling is a very blunt instrument!

Table 1: Single sampling scheme tables

Maximum producer's risk (alpha) = .05
Maximum consumer's risk (beta) = .10

AQL	LQL									
	0.010	0.020	0.030	0.040	0.050	0.060	0.070	0.080	0.090	0.100
0.002	667,3	265,2	129,1	98,1	77,1	64,1	55,1	48,1	25,0	22,0
0.004		333,3	176,2	132,2	77,1	64,1	55,1	48,1	42,1	38,1
0.006		525,6	221,3	132,2	105,2	88,2	55,1	48,1	42,1	38,1
0.008		768,10	308,5	166,3	132,3	88,2	75,2	65,2	42,1	38,1
0.010			390,7	198,4	132,3	110,3	75,2	65,2	58,2	52,2
0.012			511,10	262,6	158,4	110,3	94,3	65,2	58,2	52,2
0.014			707,15	323,8	184,5	132,4	94,3	82,3	58,2	52,2
0.016			974,22	383,10	234,7	153,5	113,4	82,3	73,3	65,3
0.018				471,13	258,8	174,6	131,5	98,4	73,3	65,3
0.020				616,18	306,10	194,7	131,5	98,4	87,4	65,3
0.022				786,24	377,13	215,8	149,6	114,5	87,4	78,4
0.024					446,16	255,10	166,7	130,6	101,5	78,4
0.026					538,20	294,12	201,9	145,7	101,5	91,5
0.028					651,25	352,15	218,10	160,8	115,6	91,5
0.030					897,32	410,18	252,12	175,9	129,7	104,6
0.032						465,21	285,14	190,10	142,8	104,6
0.034						560,26	318,16	205,11	156,9	116,7
0.036						672,32	367,19	234,13	169,10	128,8
0.038						819,40	416,22	263,15	182,11	140,9
0.040							496,27	292,17	208,13	152,10
0.042							575,32	335,20	221,14	164,11
0.044							670,38	377,23	247,16	175,12
0.046							981,58	433,27	272,18	199,14
0.048								489,31	310,21	210,15
0.050								572,37	348,24	233,17

The table gives the sample size and acceptance number: e.g. 233,17 means take a sample of 233 and accept the batch if there are 17 or fewer defectives. Blank cells indicate that AQL > LQL or $n > 999$. Quality levels (AQL and LQL) are fractions defective: the binomial distribution is used.

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Maximum producer's risk (alpha) = .05
Maximum consumer's risk (beta) = .05

AQL	LQL									
	0.010	0.020	0.030	0.040	0.050	0.060	0.070	0.080	0.090	0.100
0.002	913,4	313,2	157,1	117,1	93,1	78,1	66,1	58,1	51,1	46,1
0.004		456,4	257,3	156,2	124,2	78,1	66,1	58,1	51,1	46,1
0.006		655,7	303,4	192,3	124,2	103,2	88,2	58,1	51,1	46,1
0.008			392,6	227,4	153,3	127,3	88,2	77,2	68,2	61,2
0.010				521,9	261,5	181,4	127,3	109,3	77,2	68,2
0.012				686,13	326,7	208,5	150,4	109,3	95,3	68,2
0.014				886,18	421,10	234,6	173,5	129,4	95,3	84,3
0.016					513,13	286,8	195,6	148,5	112,4	84,3
0.018					604,16	336,10	217,7	167,6	129,5	100,4
0.020					781,22	386,12	259,9	185,7	129,5	115,5
0.022					984,29	458,15	280,10	203,8	146,6	115,5
0.024						554,19	321,12	221,9	162,7	129,6
0.026						671,24	381,15	239,10	178,8	143,7
0.028						832,31	441,18	274,12	193,9	143,7
0.030							519,22	309,14	224,11	158,8
0.032							597,26	360,17	240,12	185,10
0.034							711,32	411,20	270,14	199,11
0.036							843,39	461,23	300,16	213,12
0.038								527,27	330,18	240,14
0.040								625,33	374,21	253,15
0.042								722,39	417,24	279,17
0.044								850,47	475,28	319,20
0.046									546,33	358,23
0.048									631,39	396,26
0.050									729,46	434,29

The table gives the sample size and acceptance number: e.g. 233,17 means take a sample of 233 and accept the batch if there are 17 or fewer defectives. Blank cells indicate that AQL > LQL or n > 999. Quality levels (AQL and LQL) are fractions defective: the binomial distribution is used.

Table 2: Poisson tables

mean =	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
Probability that EXACTLY r events are contained in an interval								
r = 0	0.9048	0.8187	0.7408	0.6703	0.6065	0.5488	0.4966	0.4493
1	0.0905	0.1637	0.2222	0.2681	0.3033	0.3293	0.3476	0.3595
2	0.0045	0.0164	0.0333	0.0536	0.0758	0.0988	0.1217	0.1438
3	0.0002	0.0011	0.0033	0.0072	0.0126	0.0198	0.0284	0.0383
4	0.0000	0.0001	0.0003	0.0007	0.0016	0.0030	0.0050	0.0077
5	0.0000	0.0000	0.0000	0.0001	0.0002	0.0004	0.0007	0.0012
6	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0002
7	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
8	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
10	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
11	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
12	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
13	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
14	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
15	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
16	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Probability that r OR LESS events are contained in an interval								
r = 0	0.9048	0.8187	0.7408	0.6703	0.6065	0.5488	0.4966	0.4493
1	0.9953	0.9825	0.9631	0.9384	0.9098	0.8781	0.8442	0.8088
2	0.9998	0.9989	0.9964	0.9921	0.9856	0.9769	0.9659	0.9526
3	1.0000	0.9999	0.9997	0.9992	0.9982	0.9966	0.9942	0.9909
4	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9992	0.9986
5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998
6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Poisson probabilities

```

=====
mean =      0.9      1.0      1.1      1.2      1.3      1.4      1.5      1.6
=====
Probability that EXACTLY r events are contained in an interval
=====
r =  0   0.4066 0.3679 0.3329 0.3012 0.2725 0.2466 0.2231 0.2019
      1   0.3659 0.3679 0.3662 0.3614 0.3543 0.3452 0.3347 0.3230
      2   0.1647 0.1839 0.2014 0.2169 0.2303 0.2417 0.2510 0.2584
      3   0.0494 0.0613 0.0738 0.0867 0.0998 0.1128 0.1255 0.1378
      4   0.0111 0.0153 0.0203 0.0260 0.0324 0.0395 0.0471 0.0551
      5   0.0020 0.0031 0.0045 0.0062 0.0084 0.0111 0.0141 0.0176
      6   0.0003 0.0005 0.0008 0.0012 0.0018 0.0026 0.0035 0.0047
      7   0.0000 0.0001 0.0001 0.0002 0.0003 0.0005 0.0008 0.0011
      8   0.0000 0.0000 0.0000 0.0000 0.0001 0.0001 0.0001 0.0002
      9   0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
     10   0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
     11   0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
     12   0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
     13   0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
     14   0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
     15   0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
     16   0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000

```

```

=====
Probability that r OR LESS events are contained in an interval
=====
r =  0   0.4066 0.3679 0.3329 0.3012 0.2725 0.2466 0.2231 0.2019
      1   0.7725 0.7358 0.6990 0.6626 0.6268 0.5918 0.5578 0.5249
      2   0.9371 0.9197 0.9004 0.8795 0.8571 0.8335 0.8088 0.7834
      3   0.9865 0.9810 0.9743 0.9662 0.9569 0.9463 0.9344 0.9212
      4   0.9977 0.9963 0.9946 0.9923 0.9893 0.9857 0.9814 0.9763
      5   0.9997 0.9994 0.9990 0.9985 0.9978 0.9968 0.9955 0.9940
      6   1.0000 0.9999 0.9999 0.9997 0.9996 0.9994 0.9991 0.9987
      7   1.0000 1.0000 1.0000 1.0000 0.9999 0.9999 0.9998 0.9997
      8   1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000
      9   1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000
     10   1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000
     11   1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000
     12   1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000
     13   1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000
     14   1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000
     15   1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000
     16   1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000

```

Poisson probabilities

```

=====
mean =      1.7      1.8      1.9      2.0      2.1      2.2      2.3      2.4
=====
Probability that EXACTLY r events are contained in an interval
=====
r =  0   0.1827 0.1653 0.1496 0.1353 0.1225 0.1108 0.1003 0.0907
     1   0.3106 0.2975 0.2842 0.2707 0.2572 0.2438 0.2306 0.2177
     2   0.2640 0.2678 0.2700 0.2707 0.2700 0.2681 0.2652 0.2613
     3   0.1496 0.1607 0.1710 0.1804 0.1890 0.1966 0.2033 0.2090
     4   0.0636 0.0723 0.0812 0.0902 0.0992 0.1082 0.1169 0.1254
     5   0.0216 0.0260 0.0309 0.0361 0.0417 0.0476 0.0538 0.0602
     6   0.0061 0.0078 0.0098 0.0120 0.0146 0.0174 0.0206 0.0241
     7   0.0015 0.0020 0.0027 0.0034 0.0044 0.0055 0.0068 0.0083
     8   0.0003 0.0005 0.0006 0.0009 0.0011 0.0015 0.0019 0.0025
     9   0.0001 0.0001 0.0001 0.0002 0.0003 0.0004 0.0005 0.0007
    10   0.0000 0.0000 0.0000 0.0000 0.0001 0.0001 0.0001 0.0002
    11   0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
    12   0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
    13   0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
    14   0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
    15   0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
    16   0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000

```

Probability that r OR LESS events are contained in an interval

```

=====
r =  0   0.1827 0.1653 0.1496 0.1353 0.1225 0.1108 0.1003 0.0907
     1   0.4932 0.4628 0.4337 0.4060 0.3796 0.3546 0.3309 0.3084
     2   0.7572 0.7306 0.7037 0.6767 0.6496 0.6227 0.5960 0.5697
     3   0.9068 0.8913 0.8747 0.8571 0.8386 0.8194 0.7993 0.7787
     4   0.9704 0.9636 0.9559 0.9473 0.9379 0.9275 0.9162 0.9041
     5   0.9920 0.9896 0.9868 0.9834 0.9796 0.9751 0.9700 0.9643
     6   0.9981 0.9974 0.9966 0.9955 0.9941 0.9925 0.9906 0.9884
     7   0.9996 0.9994 0.9992 0.9989 0.9985 0.9980 0.9974 0.9967
     8   0.9999 0.9999 0.9998 0.9998 0.9997 0.9995 0.9994 0.9991
     9   1.0000 1.0000 1.0000 1.0000 0.9999 0.9999 0.9999 0.9998
    10   1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000
    11   1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000
    12   1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000
    13   1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000
    14   1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000
    15   1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000
    16   1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000

```

Poisson probabilities

```

=====
mean =      2.5      2.6      2.7      2.8      2.9      3.0      3.1      3.2
=====
Probability that EXACTLY r events are contained in an interval
=====
r =  0   0.0821 0.0743 0.0672 0.0608 0.0550 0.0498 0.0450 0.0408
     1   0.2052 0.1931 0.1815 0.1703 0.1596 0.1494 0.1397 0.1304
     2   0.2565 0.2510 0.2450 0.2384 0.2314 0.2240 0.2165 0.2087
     3   0.2138 0.2176 0.2205 0.2225 0.2237 0.2240 0.2237 0.2226
     4   0.1336 0.1414 0.1488 0.1557 0.1622 0.1680 0.1733 0.1781
     5   0.0668 0.0735 0.0804 0.0872 0.0940 0.1008 0.1075 0.1140
     6   0.0278 0.0319 0.0362 0.0407 0.0455 0.0504 0.0555 0.0608
     7   0.0099 0.0118 0.0139 0.0163 0.0188 0.0216 0.0246 0.0278
     8   0.0031 0.0038 0.0047 0.0057 0.0068 0.0081 0.0095 0.0111
     9   0.0009 0.0011 0.0014 0.0018 0.0022 0.0027 0.0033 0.0040
    10   0.0002 0.0003 0.0004 0.0005 0.0006 0.0008 0.0010 0.0013
    11   0.0000 0.0001 0.0001 0.0001 0.0002 0.0002 0.0003 0.0004
    12   0.0000 0.0000 0.0000 0.0000 0.0000 0.0001 0.0001 0.0001
    13   0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
    14   0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
    15   0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
    16   0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000

```

```

=====
Probability that r OR LESS events are contained in an interval
=====
r =  0   0.0821 0.0743 0.0672 0.0608 0.0550 0.0498 0.0450 0.0408
     1   0.2873 0.2674 0.2487 0.2311 0.2146 0.1991 0.1847 0.1712
     2   0.5438 0.5184 0.4936 0.4695 0.4460 0.4232 0.4012 0.3799
     3   0.7576 0.7360 0.7141 0.6919 0.6696 0.6472 0.6248 0.6025
     4   0.8912 0.8774 0.8629 0.8477 0.8318 0.8153 0.7982 0.7806
     5   0.9580 0.9510 0.9433 0.9349 0.9258 0.9161 0.9057 0.8946
     6   0.9858 0.9828 0.9794 0.9756 0.9713 0.9665 0.9612 0.9554
     7   0.9958 0.9947 0.9934 0.9919 0.9901 0.9881 0.9858 0.9832
     8   0.9989 0.9985 0.9981 0.9976 0.9969 0.9962 0.9953 0.9943
     9   0.9997 0.9996 0.9995 0.9993 0.9991 0.9989 0.9986 0.9982
    10   0.9999 0.9999 0.9999 0.9998 0.9998 0.9997 0.9996 0.9995
    11   1.0000 1.0000 1.0000 1.0000 0.9999 0.9999 0.9999 0.9999
    12   1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000
    13   1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000
    14   1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000
    15   1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000
    16   1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000

```


Poisson probabilities

```

=====
mean =      3.3      3.4      3.5      3.6      3.7      3.8      3.9      4.0
=====
Probability that EXACTLY r events are contained in an interval
=====
r =  0   0.0369 0.0334 0.0302 0.0273 0.0247 0.0224 0.0202 0.0183
      1   0.1217 0.1135 0.1057 0.0984 0.0915 0.0850 0.0789 0.0733
      2   0.2008 0.1929 0.1850 0.1771 0.1692 0.1615 0.1539 0.1465
      3   0.2209 0.2186 0.2158 0.2125 0.2087 0.2046 0.2001 0.1954
      4   0.1823 0.1858 0.1888 0.1912 0.1931 0.1944 0.1951 0.1954
      5   0.1203 0.1264 0.1322 0.1377 0.1429 0.1477 0.1522 0.1563
      6   0.0662 0.0716 0.0771 0.0826 0.0881 0.0936 0.0989 0.1042
      7   0.0312 0.0348 0.0385 0.0425 0.0466 0.0508 0.0551 0.0595
      8   0.0129 0.0148 0.0169 0.0191 0.0215 0.0241 0.0269 0.0298
      9   0.0047 0.0056 0.0066 0.0076 0.0089 0.0102 0.0116 0.0132
     10   0.0016 0.0019 0.0023 0.0028 0.0033 0.0039 0.0045 0.0053
     11   0.0005 0.0006 0.0007 0.0009 0.0011 0.0013 0.0016 0.0019
     12   0.0001 0.0002 0.0002 0.0003 0.0003 0.0004 0.0005 0.0006
     13   0.0000 0.0000 0.0001 0.0001 0.0001 0.0001 0.0002 0.0002
     14   0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0001
     15   0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
     16   0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000

```

```

Probability that r OR LESS events are contained in an interval
=====
r =  0   0.0369 0.0334 0.0302 0.0273 0.0247 0.0224 0.0202 0.0183
      1   0.1586 0.1468 0.1359 0.1257 0.1162 0.1074 0.0992 0.0916
      2   0.3594 0.3397 0.3208 0.3027 0.2854 0.2689 0.2531 0.2381
      3   0.5803 0.5584 0.5366 0.5152 0.4942 0.4735 0.4532 0.4335
      4   0.7626 0.7442 0.7254 0.7064 0.6872 0.6678 0.6484 0.6288
      5   0.8829 0.8705 0.8576 0.8441 0.8301 0.8156 0.8006 0.7851
      6   0.9490 0.9421 0.9347 0.9267 0.9182 0.9091 0.8995 0.8893
      7   0.9802 0.9769 0.9733 0.9692 0.9648 0.9599 0.9546 0.9489
      8   0.9931 0.9917 0.9901 0.9883 0.9863 0.9840 0.9815 0.9786
      9   0.9978 0.9973 0.9967 0.9960 0.9952 0.9942 0.9931 0.9919
     10   0.9994 0.9992 0.9990 0.9987 0.9984 0.9981 0.9977 0.9972
     11   0.9998 0.9998 0.9997 0.9996 0.9995 0.9994 0.9993 0.9991
     12   1.0000 0.9999 0.9999 0.9999 0.9999 0.9998 0.9998 0.9997
     13   1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 0.9999 0.9999
     14   1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000
     15   1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000
     16   1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000

```

Poisson probabilities

```

=====
mean =      4.1      4.2      4.3      4.4      4.5      4.6      4.7      4.8
=====
Probability that EXACTLY r events are contained in an interval
=====
r =  0   0.0166 0.0150 0.0136 0.0123 0.0111 0.0101 0.0091 0.0082
     1   0.0679 0.0630 0.0583 0.0540 0.0500 0.0462 0.0427 0.0395
     2   0.1393 0.1323 0.1254 0.1188 0.1125 0.1063 0.1005 0.0948
     3   0.1904 0.1852 0.1798 0.1743 0.1687 0.1631 0.1574 0.1517
     4   0.1951 0.1944 0.1933 0.1917 0.1898 0.1875 0.1849 0.1820
     5   0.1600 0.1633 0.1662 0.1687 0.1708 0.1725 0.1738 0.1747
     6   0.1093 0.1143 0.1191 0.1237 0.1281 0.1323 0.1362 0.1398
     7   0.0640 0.0686 0.0732 0.0778 0.0824 0.0869 0.0914 0.0959
     8   0.0328 0.0360 0.0393 0.0428 0.0463 0.0500 0.0537 0.0575
     9   0.0150 0.0168 0.0188 0.0209 0.0232 0.0255 0.0281 0.0307
    10   0.0061 0.0071 0.0081 0.0092 0.0104 0.0118 0.0132 0.0147
    11   0.0023 0.0027 0.0032 0.0037 0.0043 0.0049 0.0056 0.0064
    12   0.0008 0.0009 0.0011 0.0013 0.0016 0.0019 0.0022 0.0026
    13   0.0002 0.0003 0.0004 0.0005 0.0006 0.0007 0.0008 0.0009
    14   0.0001 0.0001 0.0001 0.0001 0.0002 0.0002 0.0003 0.0003
    15   0.0000 0.0000 0.0000 0.0000 0.0001 0.0001 0.0001 0.0001
    16   0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000

```

```

=====
Probability that r OR LESS events are contained in an interval
=====
r =  0   0.0166 0.0150 0.0136 0.0123 0.0111 0.0101 0.0091 0.0082
     1   0.0845 0.0780 0.0719 0.0663 0.0611 0.0563 0.0518 0.0477
     2   0.2238 0.2102 0.1974 0.1851 0.1736 0.1626 0.1523 0.1425
     3   0.4142 0.3954 0.3772 0.3594 0.3423 0.3257 0.3097 0.2942
     4   0.6093 0.5898 0.5704 0.5512 0.5321 0.5132 0.4946 0.4763
     5   0.7693 0.7531 0.7367 0.7199 0.7029 0.6858 0.6684 0.6510
     6   0.8786 0.8675 0.8558 0.8436 0.8311 0.8180 0.8046 0.7908
     7   0.9427 0.9361 0.9290 0.9214 0.9134 0.9049 0.8960 0.8867
     8   0.9755 0.9721 0.9683 0.9642 0.9597 0.9549 0.9497 0.9442
     9   0.9905 0.9889 0.9871 0.9851 0.9829 0.9805 0.9778 0.9749
    10   0.9966 0.9959 0.9952 0.9943 0.9933 0.9922 0.9910 0.9896
    11   0.9989 0.9986 0.9983 0.9980 0.9976 0.9971 0.9966 0.9960
    12   0.9997 0.9996 0.9995 0.9993 0.9992 0.9990 0.9988 0.9986
    13   0.9999 0.9999 0.9998 0.9998 0.9997 0.9997 0.9996 0.9995
    14   1.0000 1.0000 1.0000 0.9999 0.9999 0.9999 0.9999 0.9999
    15   1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000
    16   1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000

```

Poisson probabilities

```
=====
mean =      4.9      5.0
=====
```

```
Probability that EXACTLY r events are contained in an interval
=====
```

```
r =  0   0.0074 0.0067
     1   0.0365 0.0337
     2   0.0894 0.0842
     3   0.1460 0.1404
     4   0.1789 0.1755
     5   0.1753 0.1755
     6   0.1432 0.1462
     7   0.1002 0.1044
     8   0.0614 0.0653
     9   0.0334 0.0363
    10   0.0164 0.0181
    11   0.0073 0.0082
    12   0.0030 0.0034
    13   0.0011 0.0013
    14   0.0004 0.0005
    15   0.0001 0.0002
    16   0.0000 0.0000
```

```
Probability that r OR LESS events are contained in an interval
=====
```

```
r =  0   0.0074 0.0067
     1   0.0439 0.0404
     2   0.1333 0.1247
     3   0.2793 0.2650
     4   0.4582 0.4405
     5   0.6335 0.6160
     6   0.7767 0.7622
     7   0.8769 0.8666
     8   0.9382 0.9319
     9   0.9717 0.9682
    10   0.9880 0.9863
    11   0.9953 0.9945
    12   0.9983 0.9980
    13   0.9994 0.9993
    14   0.9998 0.9998
    15   0.9999 0.9999
    16   1.0000 1.0000
```