MATHS SHOULD NOT BE HARD: THE CASE FOR MAKING ACADEMIC KNOWLEDGE MORE PALATABLE

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Summary

This article argues that simplifying academic knowledge, and enhancing its aesthetic appeal, may be the most effective way of assisting its development and dissemination, and in the long term may be essential if humankind is to continue to progress. Eight principles for simplifying knowledge are suggested. This process may proceed hand in hand with conventional education. However, simplification can be viewed as a substitute for, or competitor of, education; there are certainly powerful forces resisting simplification within the educational community - simplifying knowledge makes it easier, which, perversely, may be seen as an undesirable lowering of standards.

Examples are given to illustrate how ideas and methods can be simplified to assist with the problems of the expert, the layperson, and the student. Most of these examples are drawn from mathematics and statistics: these disciplines are chosen because they are the basis of a very wide range of applications, and because, for many people, they symbolise what is hard about academic knowledge. The argument of the article is, however, not restricted to these disciplines.

Introduction

The title of this article was inspired by a recent leader column in the *Daily Telegraph* (16 December, 2001) which argued that "maths should be hard". I think we should aim for just the opposite, and try to make maths, and every other subject in the curriculum, as simple as possible without sacrificing the power, usefulness and elegance of the subject matter.

My aim in this article is to consider the possibility of simplifying academic knowledge, and of taking its aesthetic dimension seriously, and the implications this may have for education. Obviously, this is an ambitious goal: the title should not be taken to imply any sort of comprehensive treatment. All I want to do is to explore a possibility which is very obvious, and yet largely unrecognised, and, as far as I am aware, almost completely ignored by the educational community.

From a personal point of view, these ideas stem from my teaching. The general pattern is that I try to teach something, often without any great success. The students find it difficult, they get the answers wrong, they misunderstand both the spirit and the letter of the message, and they generally show a lack of empathy with the subject matter. So I try to improve my teaching, and the students' learning experience, and things get better - but slowly, and I gradually come to the conclusion that there is a pretty low upper limit to what we can achieve. Two explanations come to mind: the stupidity of the students, and the incompetence of the teacher. I naturally use the former to excuse the latter.

But then, sometimes, another thought occurs to me. Am I teaching the best version of the subject matter? Perhaps if it were simpler, or more aesthetically appealing, these problems would wither away? In other words, designing a more appropriate syllabus may be the best way of solving the educational problem - in the long term, if not always in the short term.

It is worth putting this into the terminology of business quality management. In the old days, quality problems were seen as the workers' fault. Nowadays, it is generally agreed to be more helpful to view them as faults of the whole system: the problem then is to redesign the system so that faults are less likely. In the educational context, knowledge is a crucial part of the system which may be redesigned. However, the usual assumption in education is that, in the last resort, it is the student - the customer, not even the worker - who is responsible for failure. In terms of quality management in business, this comes before the bad old days depicted in the historical review in the textbooks: it is an attitude which could be dubbed prehistoric.

These thoughts have occurred to me when teaching mathematical methods and statistics to non-mathematicians, research methods to business students, and in several other areas. It is not, of course, a new idea. In many contexts the importance of making things as simple and appealing as possible is taken for granted.

For example, these days word processors are easy to use. When they were first invented, however, they were complicated beasts requiring considerable effort, and training, to master. The education problem has been solved by making the task easier.

Going back a few years to Roman times, the lack of the modern notation system for numbers meant that simple arithmetic was far from simple. It is far easier to do arithmetic with numerals like 1949 than with MCMIL, and the difference is even more pronounced with fractional numbers. More recently, the advent of calculators and computers makes arithmetic trivial. We are better at arithmetic than the Romans (I am guessing here), not so much because of superior education, but because of easier methods and technology. Very similar comments apply to the adoption of the metric system of measures in place of 16 ounces to a pound, 14 pounds to a stone, and so on.

There are similar opportunities available within the current menu of academic knowledge. For example, much of mathematical statistics (*t* tests, chi square tests, analysis of variance and so on) is probability theory - very complicated but, these days, easily replaced by far simpler computer simulations (see below for more detail). The difficulty in some disciplines is mastering the jargon - there are almost always ways in which obscure terms can be made easier to assimilate. I will discuss some of the possibilities in more detail in later sections of this article. The extent of these opportunities is obviously open to debate: my point is simply that they do exist, are important, and should not be ignored.

Simplicity does have its advocates within the academic world. Cohen and Stewart (1994) make the point that "to understand something is to grasp it with your mind, to make it into an *object* that you can hold as a unit" (p. 10), and this is only possible if the something is simple enough to be grasped in this way.

Fairly obviously, there are limits to the human capacity to learn and think, and some progress has been made in describing these limits (Simon, 1996). This means that simplification is helpful if it enables knowledge to be crammed into fewer units, or "chunks" (Simon, 1996), so that humans, with their limited capacities can, almost literally, think bigger, more powerful, thoughts.

Problems, of course, can arise if this principle is taken too far. Shakespeare and Erickson (2001) refer, in approving terms, to Einstein saying

"Make everything as simple as possible. But not simpler."

In a similar vein, Stewart (1990: 215) tells the story of a report to a farmer on improving his dairy production which starts "consider a circular cow". This makes the mathematics simpler, but perhaps at the cost of losing contact with real cows. Real life may be too complex for this approach to be useful.

Complexity has recently become a fashionable concept (see Waldrop, 1994 for a popular introduction to one approach to this concept). Life is undoubtedly complex, and simple approaches which ignore this complexity are not likely to be useful. This trend has been taken further by Barnett (2000) who introduces the term "supercomplexity" which describes the situation in which we need to handle "multiple frames of understanding" as well as "overwhelming data and theories" (Barnett, 2000: 6). This leads on to a view of teaching as the "production of supercomplexity in the private space of the minds of students" (Barnett, 2000: 162).

This may be necessary, but we should surely still strive to make things as simple as possible for our students - precisely because of this supercomplexity. There is always a tension between the complexity of our situation, and the necessity to try to view it in the simplest possible way.

Simplifying something may make it less challenging for the student, but this may be necessary if the student is to cope. And those who cope easily will have the time and energy to take on new challenges, which they would not otherwise have had the opportunity to confront.

This brings us to another starting point for this article, and another reason for trying to persuade you that the ideas in it are important: the issue of the increasing quantity and complexity of human knowledge. I will discuss this in the next section, as it leads on to a more fundamental rationale for stressing ideas of simplicity and beauty than my incompetence as a teacher.

I should make it clear that my main concern is with knowledge that is learned mainly for utilitarian purposes - interpreted very broadly. These purposes might include providing a better understanding of something, or extending a theoretical framework, as well as the more practical or vocational ends of designing machinery or business processes, or of making money. This is broader than the concept of "performativity" (Lyotard, 1984; Barnett, 2000) - valuing knowledge by means of its usefulness, or impact on the world - in that it includes uses which are purely epistemological.

The only exclusion is knowledge that is learned *entirely* for its own sake. We might, for example, be interested in studying the statistical theory of analysis of variance for its own sake; to recommend that a simpler method should be studied in its place would make as much sense as recommending that mountaineers climbing Everest should climb lower mountains instead to reduce the dangers - in both cases the recommendations ignore the whole point of the enterprise. My arguments for simplicity do not, of course, apply here. Obviously, in some contexts, we may study something because it is useful, *and* because we enjoy it; the enjoyment here is obviously a good thing in all senses - this is an issue we return to in the section on aesthetics.

The argument of this article applies to a curriculum which is student-centred or teacher-centred, problem-based or conventional. My concern is with the content of the curriculum, not how it is controlled. Similarly, the word utilitarian, and the nature of the examples used, should not be taken to imply that my arguments apply only to uncritical learners. In fact, simplicity is often essential for realistic criticism; you cannot (usefully) criticise what you do not understand.

My arguments for simplicity and beauty also apply to both "Mode 1" and "Mode 2" knowledge (Gibbons et al, 1994). Mode 1 knowledge is the traditional form which conforms to the "Newtonian model" of science (p 2); Mode 2 is "transdisciplinary, ... more heterarchical and transient, ... more socially accountable and reflexive" (p. 3) than Mode 1, and furthermore, many of its practitioners work outside universities. There is a case for simplicity and beauty right across this spectrum, although the focus of my concern is the traditional Mode 1, as this sector is likely to be more hostile to these ideas.

Problems of the increasing quantity and complexity of knowledge: the expert's problem and the layperson's problem

The quantity and complexity of human knowledge is obviously increasing rapidly - one estimate is that the quantity has been doubling every seven years since the late 1960s (Midgley, 1991). Many of the implications of this are equally obvious.

The time taken to train as an expert in a particular domain is likely to increase as the boundaries of knowledge expand. Most expertise is hierarchical: for example, to understand Newton's first law of motion - $F=ma$ - you need to understand the concepts represented by the three letters in this equation, and you need to understand the interpretation of equations of this kind. Each of these concepts presupposes more basic concepts, and so on. This is the main reason why this law appears late in school physics curricula: children need time to master the prerequisites. University physics curricula include more advanced concepts which may require an appreciation of laws like $F=ma$. Obviously, as knowledge progresses, the boundaries of knowledge will expand, and the time taken to master everything on the road to reach them will increase.

This increase in training time means that experts are restricted to increasingly narrow domains, and it is becoming more and more difficult for anyone to see the broader picture as the framework of knowledge becomes more fragmented. This is likely to hinder both the development of knowledge (the expert may be unaware of other fields with insights to offer), and its dissemination and use (a comprehensive literature search on all the topics relevant to this article was not feasible, so I have certainly missed many important sources of insight and information).

In crude terms, it is useful to distinguish between the *expert's problem* of reaching the frontier of the discipline and advancing it, and the *layperson's problem* of locating and using state of the art expertise in a given domain. In practice these may merge: eg experts may have the layperson's problem in making use of other disciplines to enhance their expertise.

Commentators from a variety of perspectives have noted and commented on one or both of these problems. For example, Jacoby (2000) has linked the decline of intellectuals who write for the general public to the rise of universities - whose academics write for an increasingly specialist audience. The more fragmented and specialist this academic discourse becomes, the more difficult it becomes for outsiders to participate or even follow what is happening. Thomas Kuhn's account of normal science (Kuhn, 1970) leads to essentially the same conclusion in the context of normal science.

Conventionally, the expert's problem is treated as an educational problem: more efficient methods of learning, and longer periods in education, will cope with it. However, there are obvious limits to the time we can spend in education, and also limits to the extent to which the efficiency of the learning process can be improved. Eventually, some hierarchies are likely to develop so many steps, and the knowledge at the top will require so much pre-requisite understanding, that few, if any, will make it and progress will slow down or even stop. As Simon (1996) has pointed out, the limit is not the capacity of the human brain, but the time available for learning. (When faced with too much to learn, "no time" is a more likely excuse than "brain full".) Simon estimates that (very) approximately 50,000 "chunks" of information (Simon, 1996: 91) could be learned in a decade of training by experts in fields as diverse as chess and medicine.

The layperson's problem has drawn a more imaginative range of responses. The first of these is the development of tools for searching for expertise and information - encyclopedias, experts who act as consultants, and, more recently, web search engines. Unfortunately, lay users who manage to locate the expertise often find they then cannot understand it or use it sensibly because they lack the necessary conceptual background: eg the patient who finds the jargon on the medical website impenetrable, or the researcher who finds the right statistical technique and a package to implement it, but cannot make any sense of the output.

Another approach to the layperson's problem is the development of "popular science", and its equivalent in other areas. This includes the content of both books and other media such as television and newspapers. The aim is to provide an account of science (or philosophy or any other academic discipline) in terms which can be appreciated by the uninitiated. However, much is typically lost in the translation: experts cannot rely on popular science books on quantum mechanics, for example, because the translation is a cut-down, simplified version with far less power than the original. In addition the difficulties of translation mean that the supposedly simple version is often not very simple: eg Stephen Hawking's *Brief history of time* (Hawking, 1988) has the reputation of a best seller which is rarely actually read and understood.

And, of course, there is the educational solution: a more thorough general education, lifelong learning, and so on, should help with the problem.

However, there is another approach to the layperson's problem. This is to focus on simplifying the expert's expertise. If the expertise is simplified in such a way that it is simpler for the layperson, the layperson will then have more chance of mastering the necessary conceptual background. Furthermore, if, unlike most popular science, the simplification results in a perspective of comparable power to the original, the layperson will be in a position to share in the expert's knowledge. *If* this is possible, the layperson's problem is solved.

Furthermore, if it is also simpler from the point of view of the trainee expert, then the same approach of simplification may work for the expert's problem as well. The trainee should be able to reach the boundaries of the discipline quicker, and have more chance of extending the discipline and mastering aspects of other potentially relevant disciplines and so reducing the fragmentation of knowledge. *If* all this is possible, this would solve the expert's problem.

In some cases this may not, of course, be possible; the expert may need the more complex version. However, this is certainly not always true - no expert needs Roman numerals or old-fashioned word processors - although the professional hierarchy in charge of the institutions which control the expertise may resist this conclusion for obvious reasons.

To illustrate what I mean, I will start with some examples drawn from mathematics and statistics. These are branches of knowledge which are widely perceived as being difficult for novices, and yet are at the heart of the modern world view, so demonstrating that there are real possibilities for simplification here should help to convince doubters that the argument of this article is a practical and important one, and not a mere hypothetical possibility. This leads on to a discussion of what is meant by simplification - in fact there are a variety of possible interpretations - and of how it might apply in other fields of learning, and the implications for education and the future of human knowledge.

Simplification in mathematics and statistics

The history of science is punctuated by occasional bouts of simplification when a new, general principle is discovered which makes old approaches irrelevant. Often this new principle provides a more powerful approach, and yet one which is simpler in some sense. Classic examples are Newton's laws of motion and Darwin's work on evolution, both of which provided extremely powerful frameworks to conceptualize widely disparate aspects of reality under a single umbrella. At the leading edge, science has always valued simplicity. This is true of all sciences, but it is perhaps particularly true of mathematics, and the mathematical sciences such as statistics, because of the importance given to formulating general principles - which can only be done by simplifying and ignoring irrelevant detail. Many mathematicians would see simplicity as an essential characteristic of their discipline. This makes mathematical sciences a good illustration of my general thesis, because if these sciences could usefully be simplified, the case is likely to be stronger in other disciplines.

Simplicity is also widely regarded as a virtue a few steps back from the leading edge where "normal" scientists (Kuhn, 1970) are doing routine research and developing their disciplines. Many journal editors insist that submitted articles are as simple as possible, and approaches which emphasise simplicity are generally valued.

A few paces behind this, at the level where students in schools and universities are learning the building blocks of the discipline, the situation is rather different. These students follow the standard approaches presented to them; they are not encouraged to simplify them. At first sight, this is not a problem, because the building blocks were invented by the pioneers of the discipline who did have incentives to make things as simple as possible.

However, today's students are in a different position from yesterday's pioneers, and simplicity, like beauty (see below), lies in the eye of the beholder. Simplifications here might enable students to become experts more quickly, and laypeople to make easier sense of the expertise. Let's consider a few examples.

Much of mathematical statistics is probability theory, but *any* probability can be estimated by computer simulation (provided it can be specified in a suitable way). Computer simulation is an alternative to probability theory. Is it simpler?

To make this more definite, I will consider the estimation of a particular probability: the probability of a family of four children comprising two boys and two girls. This can be estimated by the binomial distribution of probability theory, or by computer simulation - see Wood (2001) - in either case the answer is 37.5%. Which approach is simpler?

To use probability theory you have to understand the binomial distribution, and then do a fairly simple calculation. To use the simulation method you need to see the approach to take, and then carry it out - which, of course, requires a computer.

For myself, for this example, I would say the probability theory approach is simpler. However, for more complex problems, I would prefer simulation. I regularly use simulation to check results that I cannot work out by probability theory, or where I don't have faith that my application of probability theory is correct.

On the other hand, for someone without any knowledge of probability theory, the simulation method would, I think, be simpler for the four children problem.

More generally the question is which approach should we encourage students (and other users) to adopt? The main criterion I would advocate for this choice is the overall simplicity of each approach and the curriculum necessary to support it - provided, of course - that each alternative offers a similar level of power in terms of its ability to solve problems. Probability theory is a very complex branch of mathematics, and most of its results only apply under a restricted range of conditions. Against this, computer simulation approaches can be used without any knowledge of the superstructure of probability theory (standard deviations and the normal distribution are two of the lower rungs on this ladder), and, in general, the results are valid under a far wider range of conditions (Simon, 1992; Davison and

Hinkley, 1997). It is possible to get answers, and understand their rationale, with a tiny fraction of the background knowledge required by probability theory. In this sense the computer simulation approach is far simpler.

Solving equations, and finding the maximum or minimum value of a function, are important problems for mathematics. These days, spreadsheet "optimisers", or "solvers", can be used to find answers to many of these problems - not always the exact answer, but usually close enough for practical purposes. These spreadsheet tools perform, in effect, computer assisted trial and error, which has many of the same advantages as computer simulation of probabilities: the main one being that one easy method solves everything. (There are also a few disadvantages - see Wood, 2001, for a discussion of the pros and cons.)

The principle behind these methods is that the power of a computer is used to make crude, iterative methods viable and powerful. I have called methods based on this principle *crunchy methods* - see Wood, 2001 for a discussion of these, and of their advantages and disadvantages.

A second method of simplification is *conceptual reengineering* (the association with business process reengineering - Hammer, 1990 - is deliberate because many of the same issues arise in both contexts): redesigning some of the concepts so that they are more in tune with potential users' background knowledge. Many of the concepts of statistics have evolved for their mathematical convenience - it is often easy to switch to more user friendly concepts - eg the quartiles of a distribution provide a more user friendly handle on the idea of the spread of a distribution than do the standard deviation and the variance (Wood et al, 1998).

Similarly, algebraic notation is a foreign langauge for many people, as is the notation system used for formulae in spreadsheets. Unfortunately these two notation systems are different - there would be a strong case for using the same system in both contexts.

Another example occurred to me teaching reliability theory to a class on an MSc programme. The functions e^x and *log x* (the natural logarithm of x) on most calculators are two functions for analysing growth at a constant proportional rate. They appear in formulae used in many contexts - eg biology, economics, finance, as well as reliability analysis. Few people (including most of the students in my group) understand much about these functions because this understanding requires a thorough grasp of calculus and the theory of logarithms. However, by slightly redefining the concepts, it is possible to provide a direct link to common sense and bypass logarithms and calculus completely (see the Excel spreadsheet at http://www.pbs.port.ac.uk/~woodm/cgandsg.xls). The revised functions are by no means trivial: they require some hard thought, but it is not necessary to master calculus and the theory of logarithms.

These two principles - adopting crunchy methods and conceptual reengineering - afford enormous opportunities to simplify the curriculum in mathematics and statistics. This has the potential to benefit both laypeople and experts - although we should obviously bear in mind the different needs of the two groups.

However, in practice, the inertia of the conventional approach is so strong that these opportunities are almost completely ignored. I am sure that most of my colleagues would dismiss most of this section as unworthy of serious consideration. I will consider the reasons for this after looking at the opportunities for simplification in other areas, and also at the importance of the aesthetic dimension.

Suggested principles for simplifying knowledge

Simplicity refers to the property of being "easily understood or done" (*Oxford Compact English Dictionary*, OUP, 1996), which suggests two aspects which Ward (1989) calls "transparency" and "constructive simplicity". Crunchy methods such as computer simulation may be more transparent, but they may also be more difficult to "do" than conventional methods. The balance of advantages clearly depends on the expertise of the learner/user, the available technology and so on. I will not discuss these issues in detail here; instead I will look at a number of approaches which may deliver simplicity in either of these senses, or in both.

We have mentioned the switch to *crunchy methods* and *conceptual reengineering* as two possibilities. These are not, of course, independent: switching to simulation methods from mathematical probability theory will inevitably lead to a new set of concepts. I would like to suggest six further principles, which may be useful - singly or in combinations - to simplify academic knowledge. These are the principles of *ignoring history*, *shallow hierarchies*, *generalisation*, *minimum value*, *using aids for processing and memory*, and *just-in-time learning*.

Students of many disciplines (typically those which have not evolved to be a "normal science" in Kuhn's (1970) terminology) are taught about a variety of different, alternative, approaches to problems. If, say, a student is writing a dissertation on some aspect of quality management in a particular organisation, then a survey of possible approaches would be expected. This is despite the fact that there may be a clear winner as far as the best approach for the organisation in question is concerned. Paying homage to other authors is part of the academic game. This is useful in so far as it encourages flexibility, and an openness to differing ideas, but if it becomes the equivalent of studying the history of the discipline as well as the discipline, it may represent an unhelpful degree of extra complexity for the student.

In a normal science (Kuhn, 1970), the situation is rather different. Here, students are likely to be taught an edited version of history, leaving out the bits which led up to dead ends. The point here is that there may be a more efficient approach for current students - such as the possibilities discussed in the section above.

In either case there may be a case for *ignoring history*, or at least ensuring that the history which is studied is studied for a purpose.

The second principle is the principle of *shallow hierarchies*. To understand the theoretical approach to the four children problem (see above) you need to understand about the laws of probability, about the mathematical idea of "combinations", and also the prerequisites for understanding these. This understanding is hierarchical and there are at least three levels in the hierarchy. More complex probability theory will give rise to hierarchies with many more levels. On the other hand, computer simulation just involves a single principle, which follows from the idea of probability and how it can be simulated on a computer. The issue here is the depth of the hierarchy. This may be difficult to define unambiguously in numerical terms, but there is little doubt that some learning hierarchies are shallower than others, and that the shallow ones have massive advantages in terms of simplifying the educational process.

This is related to the principle of *generalisation*. Other things being equal, a general framework covering a broad area is likely to be simpler than a number of different frameworks, each of which only works in a limited domain. For example, computer simulation can do the work of the binomial, hypergeometric and Poisson distributions of traditional probability theory; one principle has replaced three. (In practice, this example, and others similar to it, are more complex than this summary may suggest. On the one hand, using the general principle is by no means trivial - practice is required - and on the other, the scope of the simulation approach is far broader than these three distributions.)

The next principle is that of *minimum value*. This is simply the suggestion that if some aspect of an academic discipline gives only a slight advantage over untutored "common" sense, by whatever criteria are deemed relevant - access to truth, insight, practical usefulness or whatever - then this slight advantage may not be worth the effort of getting to grips with relevant aspects of the discipline. There is a minimum value below which it is not worth the bother. It may be more sensible to spend the time thinking the problem out for oneself, than consulting the works of academic experts.

As with the last principle, a precise, numerical, estimate of such a minimum value is not likely to be forthcoming, but the acceptance that such a minimum value exists seems a useful principle.

The sceptic may wonder whether some aspects of some disciplines have zero or negative value; their study would be ruled out by this criterion. For example, technical disciplines, such as statistics, understood on a surface level, may only succeed in interfering with common sense, and may thus have a negative value to such a user.

The obvious *aids for processing and memory* are computer aids. These enable more powerful methods to be used than would otherwise be possible, and also relieve the learner of the necessity of following the detail of what, for example, a web search engine, or a computer statistics package, is doing. The computer aids can be treated as "black boxes" (Wood et al, 1997) - a tactic which has potential dangers as well as enormous benefits (Wood et al, 1997).

It is worth remembering, however, that this process started with the invention of writing, which can be used as an "external memory". All these devices simplify the learner's task in obvious ways - as part of the task is automated, or contracted out to an external device.

Just-in-time learning refers to the tactic of not learning something until it is needed

(Wood et al, 1997). Conventional education involves learning things just-in-case they are needed. This inevitably means that a large amount has to be learned years in advance of its potential use; almost inevitably much that is learned will turn out to be inappropriate or will be forgotten. Switching to just-in-time learning has obvious advantages in terms of reducing these difficulties. If it is to be viable, however, the methods to be learned need to be simple, and the learner needs an overview of the subject area that will enable relevant areas to be picked out just-in-time for learning.

The other side of the coin, when considering any of these principles, is the power available to the user or learner. A layperson with a pop science understanding of quantum mechanics is not in a position to do much with his understanding; on the other hand the layperson with a simulation approach to statistics has a very powerful tool at her disposal.

The importance of aesthetics

As discussed above, simplicity refers to ease of understanding, and use. The quest for simplicity tackles one of the obstacles to a full appreciation of academic knowledge - that of the often overwhelming complexity. The other problem is motivation. There are two possible types of motivation. Firstly, people may be motivated to try to learn an academic discipline because they feel that it will lead to a greater understanding of important issues, or to more fruitful decision making, or simply to passing an examination. In all cases, the motivator is in the future, and extrinsic to whatever is being learned.

The other type of motivation is intrinsic to the academic discipline. People learn something because they enjoy it, because it's fun, because they think it's beautiful. Many of the great pioneers of science were not motivated by money or the prospect of the power improved knowledge would bring, but by these aesthetic factors (see, for example, Poincare, 1981; Hardy, 1940, sections 10-11).

Obviously, if this is possible, if things can be understood in a way which is fun or beautiful, then this is obviously a good thing. Our studies may then be motivated both by our enjoyment of the subject, and by the prospect of its usefulness.

The difficulty, of course, is that beauty lies in the eye of the beholder. An equation which seem elegant, profound and beautiful to a trained mathematician, may seem incomprehensible and ugly to a layperson. As far as I am aware, the study of the aesthetics of academic disciplines from the perspective of the uninitiated is not a thriving area of research. Perhaps it should be?

The conservative influence of education

Any large, institutionalised system has an inertia which makes change difficult. This inertia may be due to ingrained habits and assumptions, the vested interests of practitioners and other stakeholders, and the various standards and documents used by the institutions (textbooks and standards relating to particular disciplines), and so on. The education system is also part of the power structure of society: educational institutions have an important role in handing out the certificates which are necessary to progress in many contexts - which means that change is unlikely to be welcomed if it is perceived to interfere with this function of education.

In addition to all of these factors, education has two further features which make it particularly hostile to the possibility of simplifying the curriculum by means of any of the principles discussed above.

The first is the idea of academic "standards": the assumptions that academic knowledge is difficult, that these difficulties must be faced rather than avoided, that a considerable degree of hard work and intelligence is necessary to succeed in this, and that rigorous assessment is essential. Students who are inadequate in these respects will fail these assessments; such failures are the inevitable concomitant of academic standards. In the words of a recent leader column in the *Daily Telegraph* (16 December 2001), "maths should be hard".

These assumptions are buried deep in the framework of assumptions on which education rests. To challenge these assumptions is a heresy. If standards slip, then what remains of education is of little value - this is, to most educationalists, completely obvious.

In fact, the whole argument of this article challenges these assumptions. If we can make academic knowledge easier, so that difficulties are avoided, less intelligence and work is required to succeed, and failure is less likely, then this is surely a good thing. If we are learning to drive a car, then devices such as automatic gearing which make the task easier are to be welcomed - provided the new driver is not likely to be faced with a non-automatic car to drive later.

Academic standards - keeping things difficult - are of obvious benefit to teachers, who might be out of a job if things were too easy to require a teacher, and to experts, whose expertise might be devalued if it were simpler. There are strong vested interests in favour of the ideology of academic standards.

The second feature of education which makes it hostile to the quest for simplicity is the fact that the task of education is viewed as assisting learners to learn a *given* body of knowledge. This body of knowledge is treated as fixed: the creation, discovery or development of this body of knowledge is a task for another team - the research team. Educationalists put over what researchers produce; they do not change it in any way. This means there is no opportunity to adjust the body of knowledge according to the needs of the learner. The quest for simplicity and beauty needs this feedback from the customer in the design of the product. The education system which sells a product with no attention to customer feedback is in a similarly unhealthy position to a commercial manufacturer which does not undertake any market research to find out what people want and what they are able to use.

These factors mean that, although the education system has sympathy with the student's problem - the knowledge to be learned seems hard and unappealing, and there are rigorous assessments which may lead to eventual failure - there is very little sympathy with the obvious solution - making the work easier and more fun.

There are a few corners on the edge of academia where this is not true, where simplicity is valued. One such corner is management consultancy (Ward, 1989), where a simple approach is valued because managers tend to lack time and energy to follow complex approaches. The reason for this is that the product of interest here is not an abstract academic product, but real, practical usefulness.

There is a sense in which simplification is a substitute for education. Either you attend college and get formal qualifications in the traditional manner, or you turn to books, the media or the purveyors of Mode 2 knowledge (Gibbons et al, 1994) for a simplified, but still useful version.

Conclusions

This article puts the case for:

* viewing academic knowledge, not as fixed and immutable, but as something which can, and should, be adapted to the needs of users and learners;
* striving for simplicity and beauty, from the perspective of users or learners, when

developing appropriate curricula: eight principles for simplifying knowledge are suggested.

Simplification from the perspective of trainee experts should enable expertise to be acquired more quickly, which may in turn enable experts to develop their subject further, or to develop a broader expertise, than would otherwise be possible. This helps to solve the expert's problem.

Simplification from the point of view of the layperson has similar benefits. At present, much of mathematical and statistical knowledge is not well integrated into the general culture, or world view, because it is too complex to be understood. This clearly impoverishes the culture; the drive for simplification is the logical approach to the layperson's problem. Ideally, the layperson's simplification would correspond with the expert's, but this may not always be possible.

Furthermore, simplifying the curriculum, and making it more appealing, should help with the student's problem - the fact that many students do not enjoy their studies because they take too much time and effort.

In this article I have discussed some of the possibilities, and also given a few examples. I suspect, however, that some readers will feel the examples are at best unproven, and the argument of this article is of little, if any, practical importance.

The possibilities are largely unproven, and they are indeed difficult to prove, because the assumptions of the education system (explained in the section above) make it difficult to take seriously the possibility of substantial simplification of academic knowledge. In my view it would be possible to simplify the mathematics and statistics taught in secondary schools, and as tools in university courses for other areas of the curriculum, by a factor of at least two. If this is even partly true, the millions of hours every year spent learning mathematical and statistical methods, all around the globe, could be devoted to other purposes, or the time could be spent developing a more powerful understanding of these areas. Particularly in places where education is a scarce luxury, this is a possibility which should surely not be ignored.

In a wider context, in the future, the problems of developing and disseminating knowledge can only get worse. I believe that the quest for simplicity and beauty at all levels of the hierarchy of knowledge will become a necessity. Otherwise, progress will slow down, or even cease, because there is simply too much to keep up with.

Ivan Illich predicted, in a book first published in the late sixties, that "by the end of this century, what we now call school [which includes universities] will be a historical relic, developed in the time of the railroad and the private automobile and discarded along with them." (Illich, 1973: 109). None of these predictions has proved accurate, and it may well be that universities will survive in their present form to the end of the present century, testing and failing students on increasingly complex, ugly and useless fields of learning. The pressures resisting change are strong, but I hope they are breached sooner rather than later.

References

Barnett, R. (2000). Realizing the university in an age of supercomplexity. Buckingham: SRHE and Open University Press.

Cohen, J., & Stewart, I. (1994). The collapse of chaos: discovering simplicity in a complex world. New York: Penguin.

Davison, A. C.; Hinkley, D. V. (1997). Bootstrap methods and their application. Cambridge: CUP.

Gibbons, M., Limoges, C., Nowotny, H., Schwartzman, S., Scott, P., & Trow, M.

(1994). The new production of knowledge: the dynamics of science and research in contemporary societies. London: Sage.

Hammer, Michael. (1990, July). 'Reengineering work: don't automate, obliterate.' Harvard Business Review.: 104-112.

Hardy, G. H. (1940). A mathematician's apology. Cambridge: Cambridge University Press.

Hawking, Stephen W. (1988). A brief history of time: from the big bang to black holes. London: Bantam Press.

Illich, I. (1973). Celebration of awareness. Harmondsworth: Penguin Education.

Jacoby, R. (2000). The last intellectuals: American culture in the age of academe. : Basic Books.

Kuhn, T. S. (1970). The structure of scientific revolutions (2nd edition). Chicago: University of Chicago Press.

Lyotard, J. (1984). The postmodern condition: a report on knowledge (translated from French by Geoff Bennington and Brian Massumi). Manchester: Manchester University Press.

Midgley, M. (1991). Wisdom, information, and wonder: what is knowledge for? London and New York: Routledge.

Poincare, H. (1981). 'Mathematics as aesthetic creation.' in S. Brown, J. Fauvel, & R. Finnegan (eds), Conceptions of inquiry, (pp. 12-21). London: Methuen.

Shakespeare, T., & Erickson, M. (2001). 'Different strokes: beyond biological determinism and social constructionism.' in H. Rose, & S. Rose (eds), Alas, poor Darwin: arguments against evolutionary psychology, (pp. 190-205). London: Vintage.

Simon, Herbert A. (1996). The sciences of the artificial. Cambridge, Massachusetts: MIT Press.

Simon, Julian L. (1992). Resampling: the new statistics. Arlington, VA: Resampling Stats, Inc.

Stewart, I. (1990). Does God play dice? London: Penguin.

Waldrop, M. M. (1994). Complexity: the emerging science at the edge of order and chaos. London: Penguin.

Ward, S. C. (1989). 'Arguments for constructively simple models.' Journal of the Operational Research Society, 40(2): 141-153.

Wood, M., Cahill, P., & Hicks, J. (1997). 'Computer packages as cognitive paradigms: implications for the education of accountants.' Journal of Accounting Education, 15(1), 53-69.

Wood, M., Capon, N., & Kaye, M. (1998). 'User-friendly statistical concepts for process monitoring.' Journal of the Operational Research Society, 49(9), 976-985.

Wood, M. (2001). 'The case for crunchy methods in practical mathematics.' Philosophy of Mathematics Education Journal (a web journal at http://www.ex.ac.uk/~PErnest/), 14 (http://www.ex.ac.uk/~PErnest/pome14/wood.htm)